Cluster States in Measurement Based Quantum Computing

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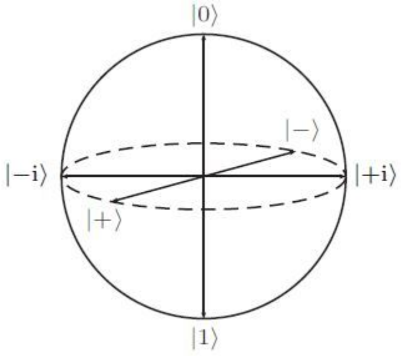
The Circuit Model

Prepare a product state of a few qubits, computation proceeds by application of unitary transformations (rotations) over the qubits, measure in some basis.



Qubits are represented as vectors on the Bloch Sphere

X Y Z

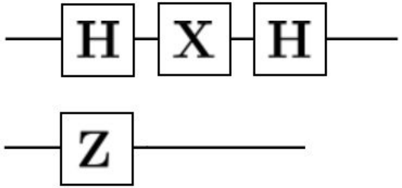
CZ CNOT

Unitaries are length preserving transformations over the Bloch Sphere

$$\{|0\rangle, |1\rangle\}$$
$$\{|+\rangle, |-\rangle\}$$
$$\{|i\rangle, |-i\rangle\}$$

Measurements are inherently probabilistic

A few Examples



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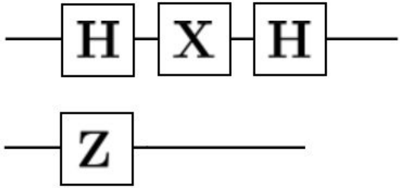
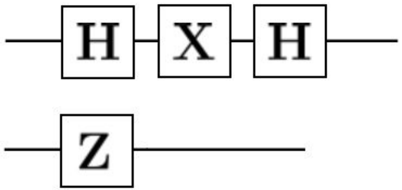
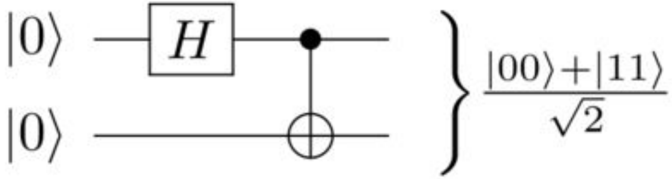


The Hadamard Gate change bases from X to Z and vice versa

A few Examples





The Hadamard Gate change bases from X to Z and vice versa

The output may not always be expressed as a product state

This is Entanglement

$$|B_{00}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$
$$|B_{01}\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle - |11\rangle\right)$$
$$|B_{10}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$
$$|B_{11}\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

The 4 entangled states are called the Bell States.

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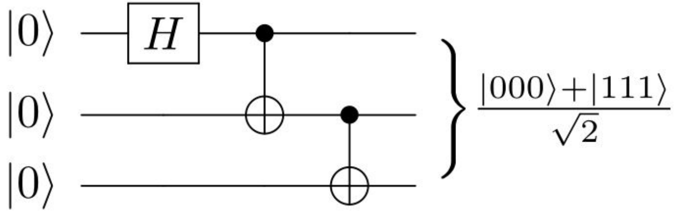
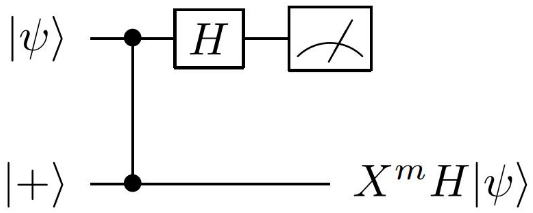


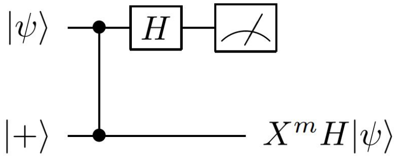
Entanglement in multiple qubits too is easily demonstrated.

Single Qubit Teleportation



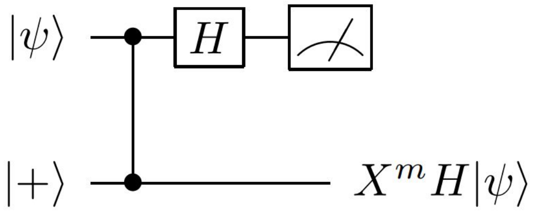
Single Qubit Teleportation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



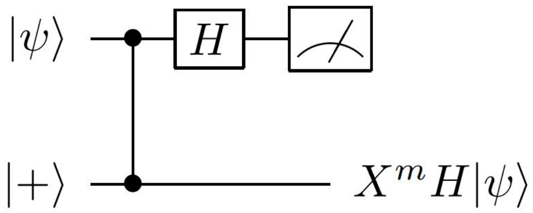
$$\begin{aligned}
CZ_{12}|\psi\rangle_1|+\rangle_2 &= \alpha|0\rangle_1|+\rangle_2 + \beta|1\rangle_1|-\rangle_2 \\
&= \frac{1}{\sqrt{2}}|+\rangle_1(\alpha|+\rangle_2 + \beta|-\rangle_2) + \frac{1}{\sqrt{2}}|-\rangle_1(\alpha|+\rangle_2 - \beta|-\rangle_2) \\
&= \frac{1}{\sqrt{2}}|+\rangle_1 H(\alpha|0\rangle_2 + \beta|1\rangle_2) + \frac{1}{\sqrt{2}}|-\rangle_1 HZ(\alpha|0\rangle_2 + \beta|1\rangle_2) \\
&= \frac{1}{\sqrt{2}}|+\rangle_1 H|\psi\rangle_2 + \frac{1}{\sqrt{2}}|-\rangle_1 HZ|\psi\rangle_2 \\
&= |m\rangle_1 \; X^m H|\psi\rangle_2
\end{aligned}$$

Single Qubit Teleportation



Here randomness manifests itself through the X^m gate

Single Qubit Teleportation



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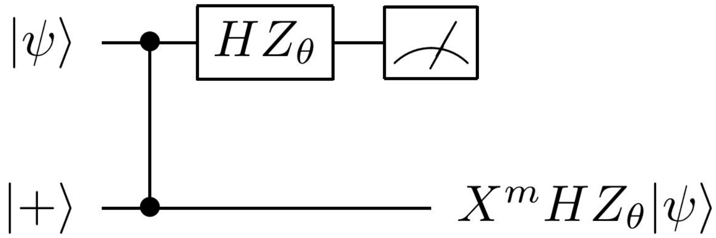
The Big Idea: We can apply a transformation by simply changing our basis for measurement.



Nielsen, Cluster State Quantum Computation. 2005

The Single Qubit Gate

Any unitary transformation can be decomposed as rotations around its euler angles, that is

$$U = R_x(\zeta) R_z(\eta) R_x(\xi)$$

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What happens when you simply do the 3 rotations?

$$X^{m_3} H Z_{\theta_3}\ X^{m_2} H Z_{\theta_2}\ X^{m_1} H Z_{\theta_1} |\psi\rangle$$

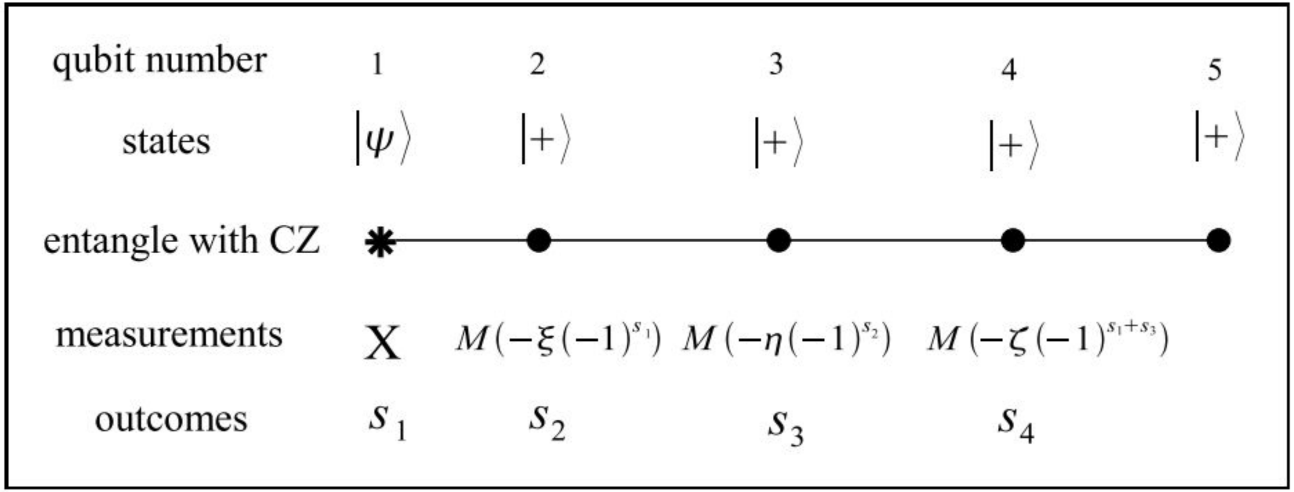
The Single Qubit Gate

Any unitary transformation can be decomposed as rotations around its euler angles, that is

$$U = R_x(\zeta)R_z(\eta)R_x(\xi)$$

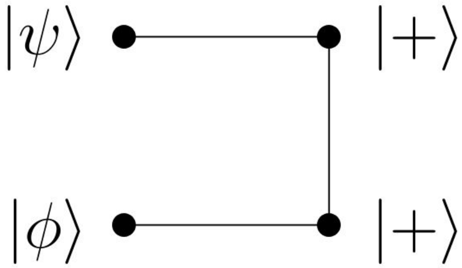
Next Big Idea: Adapt measurements for subsequent operations, based on outcomes from previous measurements

The Single Qubit Gate



Jozsa, An introduction to measurement based quantum computation, 2005

The CZ Gate



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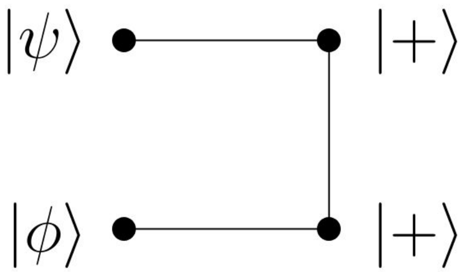
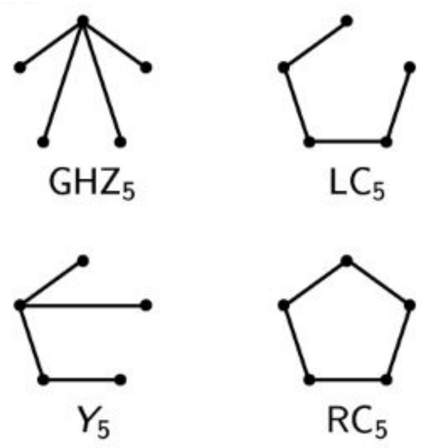


The CZ gate together with rotation in arbitrary axes is enough to establish the universality of MBQC.

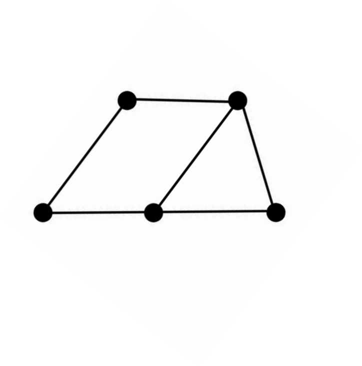
All of this just with single qubit measurements, parallely and adaptively.

Graph states

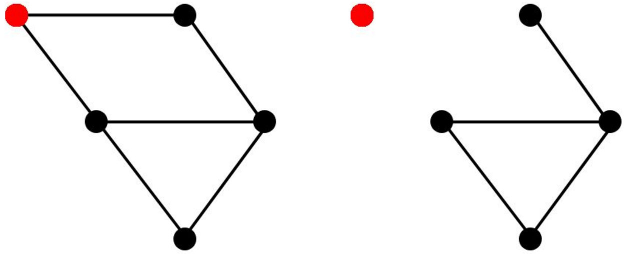
The entire scheme and its logic can be written out as a graph where the vertices are + states and edges are CZ gates.



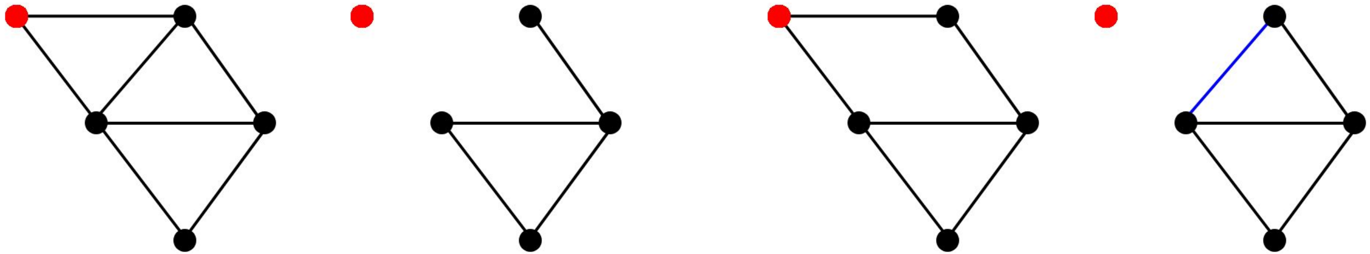
$$|G\rangle = \prod_{\text{all edges}(a,b)} C_z^{ab} \, |+\rangle^{\otimes \text{all vertices}}$$



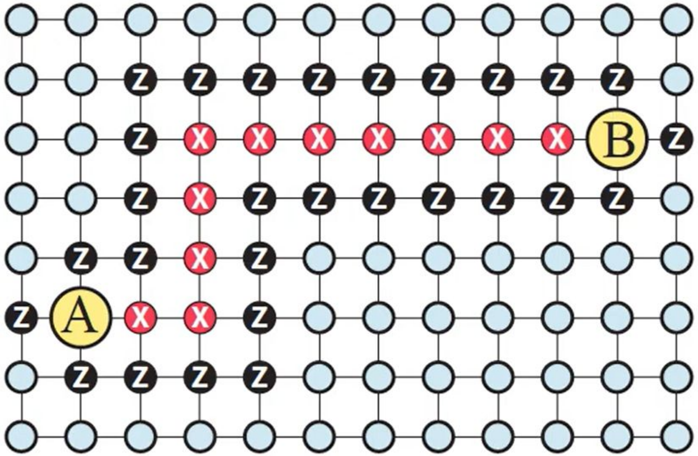
Applying a Z measurement removes all adjoining edges



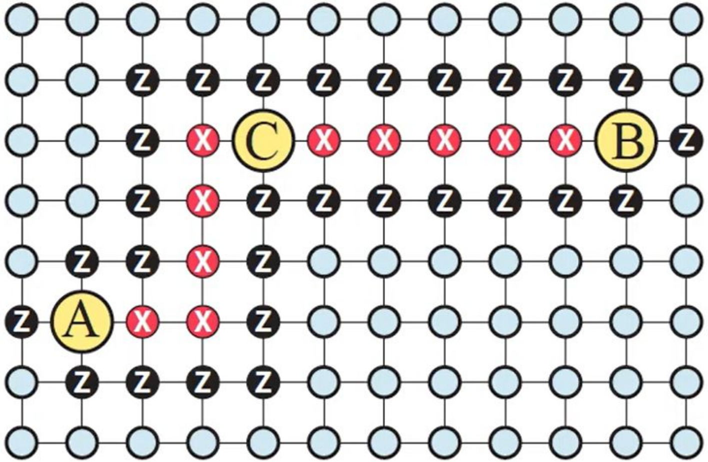
Applying a Y measurement



Applying an X measurement propagates the entanglement



Bell State between A, B

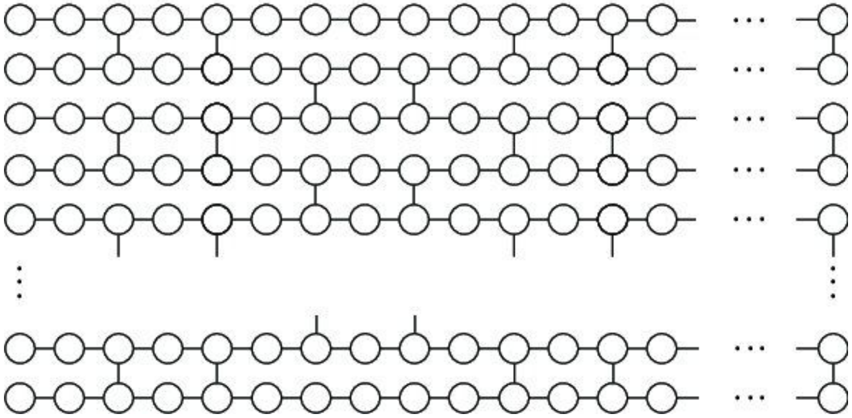


GHZ State between A, B and C

Slides from Raussendorf, MBQC Intro, 2021

Consider the Brickwork State

Computation proceeds along each row, logic is established through the choice of base along the horizontal.



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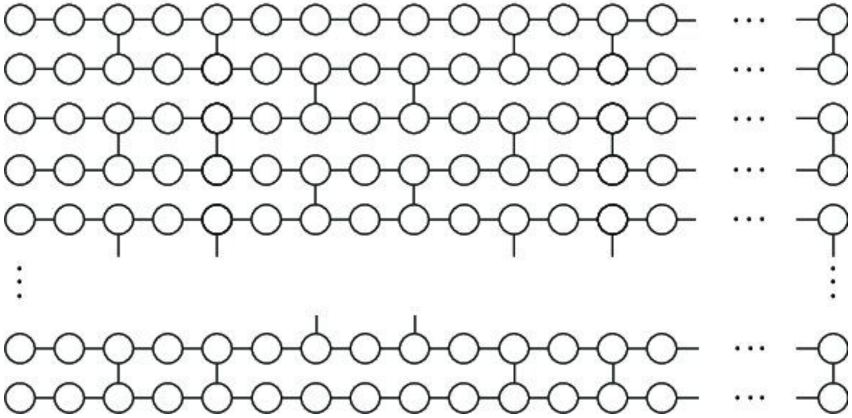
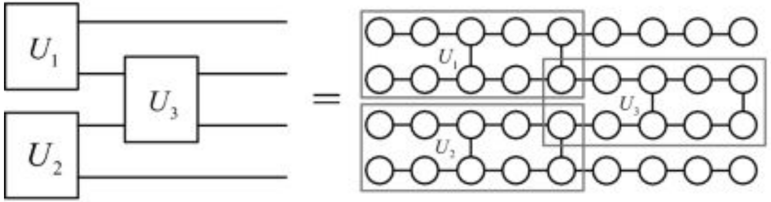
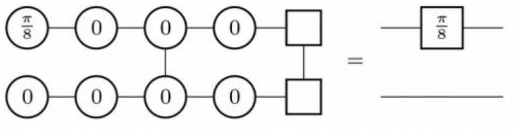
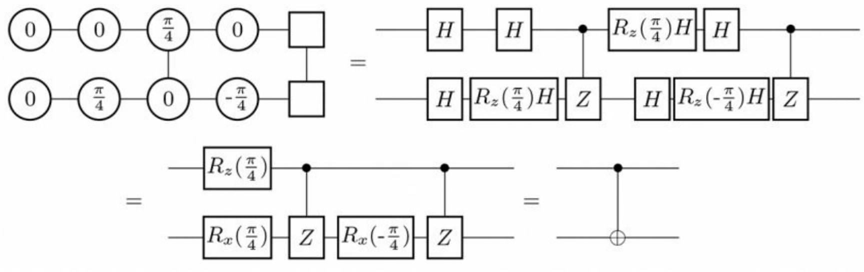




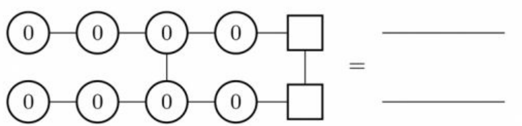
Yang, Bai, Quantum Information Processing, 2022



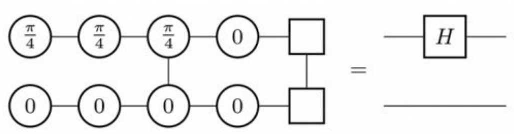
Implementation of a $\pi/8$ gate



Implementation of a CTRL-X



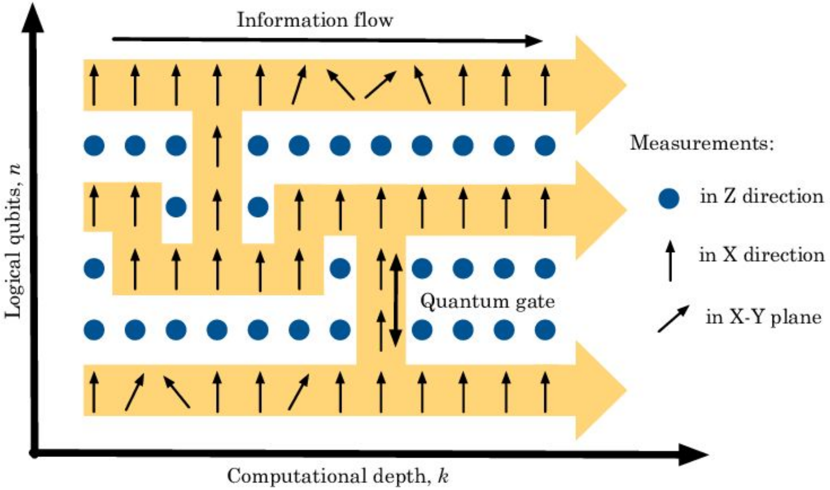
Implementation of the identity



Implementation of a Hadamard gate

Kashefi, Wallden, Garbled Quantum Computation, 2017

Implementations



Raussendorf et al, MBQC on cluster states, 2003

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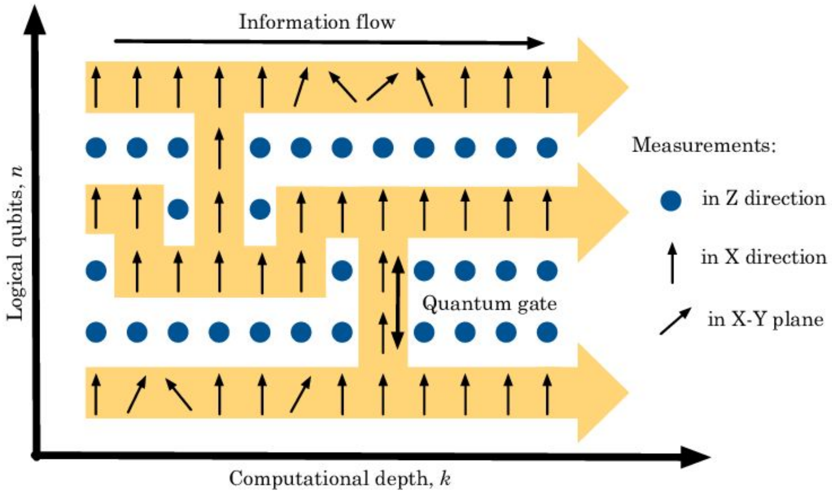

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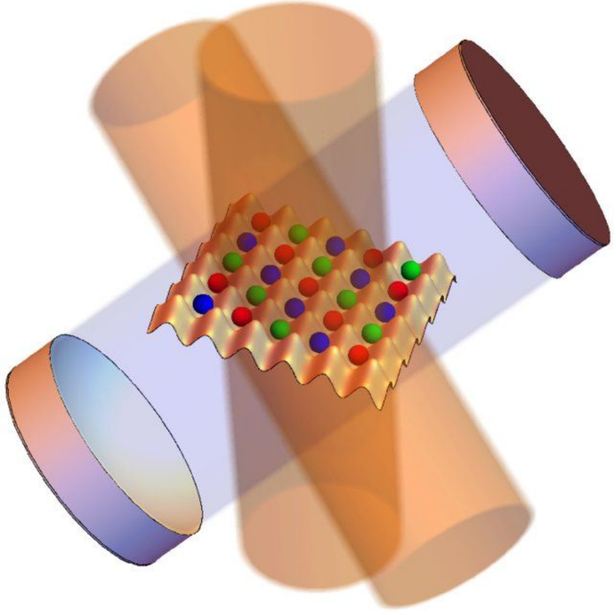

Santiago et al, PRA 93(6), 2016

References and further reading

1. *Measurement-based quantum computation on cluster states,* Raussendorf, Browne, Briegel, PRA 68
2. *An introduction to measurement based quantum computation,* Jozsa, arXiv:quant-ph/0508124
3. *One-way Quantum Computation*, Browne and Briegel, arxiv:quant-ph/0603226
4. *Cluster-state quantum computation*, Nielsen, 2005
5. *Computation by measurements: A unifying picture*, Aliferis and Leung, PRA 70

Thank you for your time.